Balancing Plate

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Abstract

The purpose of this project is to design and fabricate an initially unstable system then implement a feedback control loop to make it stable. We built a plate which ballances a ball in two dimensions using a web camera to determine the location of the ball. That position is input into a discrete lead controler then translated to displacements of servo motors which control the angle of the plate. The plate balances the ball and disturbs rejections with a reasonable transient. Since it is a type two system, it is still stable for force inputs on the ball with some finite steady state error.

Introduction

Goals

In order for our system to be considered a success it must meet certain performance specifications. Firstly, it must be able to ballance a ball on the plate. It must also be able to reject disturbances of the ball. The system should also reject those disturbances with a diecent transient response, on the order of a second or two.

The Plant

Mechanical Design

Two servos actuate the motion of the plate. The plate is supported by a ball joint in one corner and the two servos are mounted orthagonal to eachother to alter the angles of each axis independently. The servos have an arm which has a ball joint on the end bolted to a bracket which is bolted through the plate. Those ball joints are necessary to prevent bending in the bracket or arm when the other servo travels through its range of motion.

The angular displacement of each servo causes an angular displacement of the table. These displacements were measured by an inclinometer for several servo positions to create a table of values. We used a quadratic regression

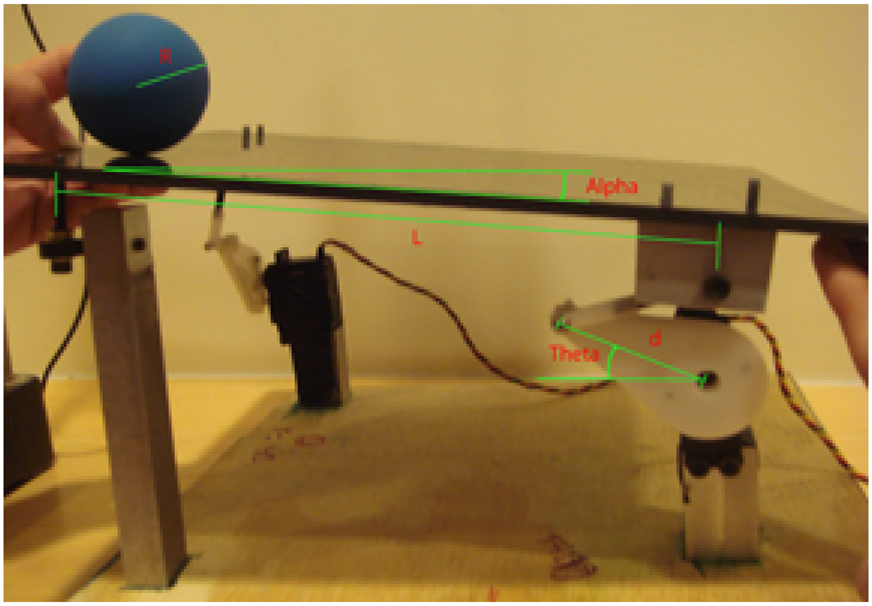

Figure 1: The Physical System

to yield a table of values. That table of values is referenced by the code to determine what motor position to use for a desired angle of the plate.

Transfer Function

The input in our system is the position of the ball. The parameter we can control is the angle of the plate. We then derived the transfer runction from x displacement of the ball to angular displacement of the table. Given the small angle approximation, the governing equation for the angle of the plate and the position of the ball is

$$(\frac{J}{R^2} + M)\ddot{x} = -mg\alpha$$

We approximate the angle of the plate to be proportional to the angle of the servo for our simulation.

$$\theta\frac{d}{L} = \alpha$$

Taking the laplace transform of the governing equation we find

$$\frac{X(s)}{\theta(s)} = \frac{-mgd}{s^2 L(\frac{J}{R^2} + M)}$$

This simplifies to two poles at the origin with a constant gain

$$\frac{X(s)}{\theta(s)} = \frac{K_{plant}}{s^2}$$

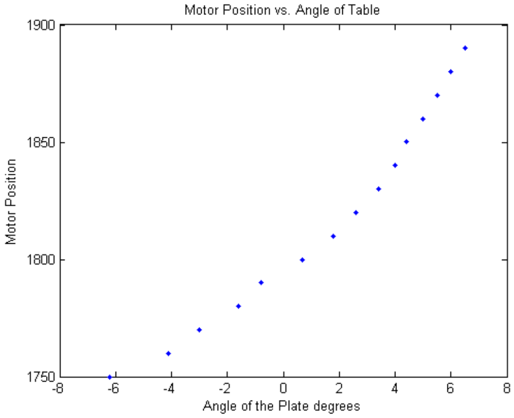

Figure 2: Angular displacement of table vs. Motor position. The curve is modeled by a quadratic regression.

The time delay in our system is also simulated. We measured the delay then multiplied our plant transfer function by

$$H(s) = e^{-\tau s}$$

Our uncompensated loop transfer function then becomes

$$L(s) = e^{-\tau s}\frac{K_{plant}}{s^2}$$

Lead Compensator

A lead compensator is implemented in order to move the poles off the origin and make the system stable.

$$G_c(s) = K\frac{\alpha\tau s + 1}{\tau s + 1}$$

alpha is set to a value of 10 to achieve a positive phase bump of fifty five degrees. τ is then set baised on our desired crossover frequency. We chose a crossover around 2 rad/s which gave 49 degrees of phase margin and prevented us from the effects of our motor dynamics.

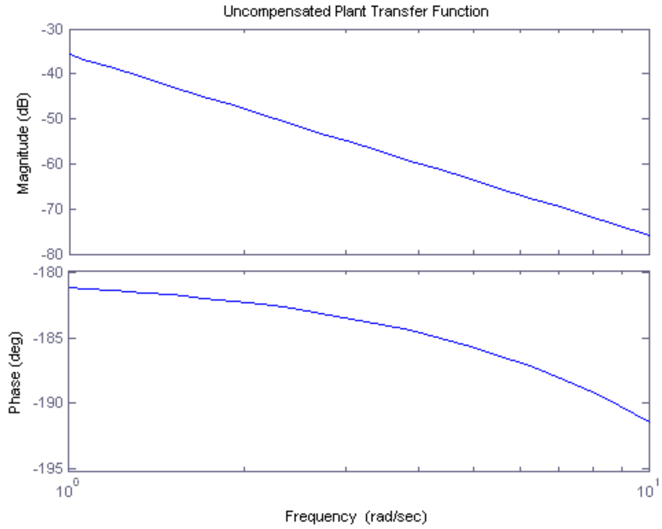

Figure 3: The uncompensated loop transfer function. Note the phase is always less than -180 degreese revealing that the system is always unstable for any gain.

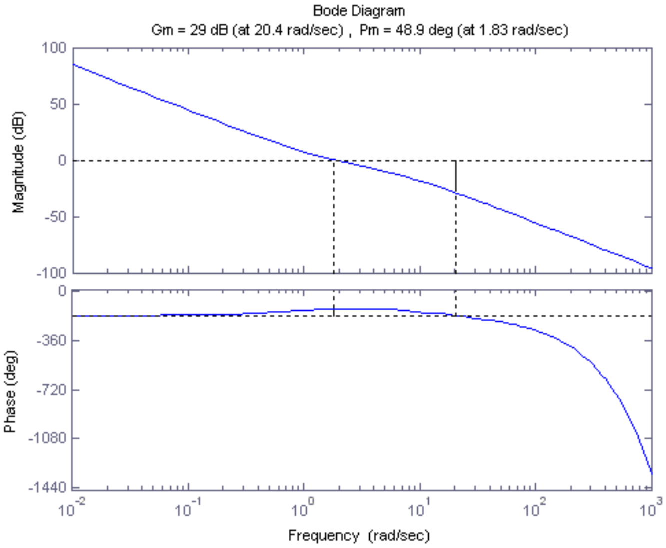

Figure 4: Compensated loop transfer function. Note the positive phase of the lead compensator makes the system stable for certain gain values.

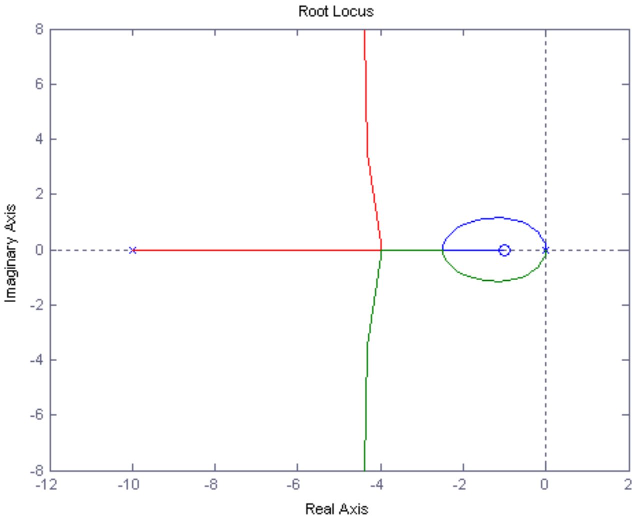

Figure 5: The two poles come off of the origin, move along the real axis and break off again. We witnessed this behavior in tuning the system. For some gain values the system had overshoot and others it did not.